Impulsive Noise Reduction in Natural Images by Plane and Paraboloid Regression

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Abstract

Presenting a clear image by reducing the noise to a minimal level is one of the most fundamental research topics in image processing. Different types of noise are initiated during the process of acquisition to digitization of an image, causing degradation in quality. As there is no way for total elimination, several methods are employed depending on the type of noise. In this paper, we present a method based on least squares regression analysis to detect and eliminate impulsive noise from an image. A sweeping window of certain dimension calculates how well a general plane fits over the pixels currenly inside the window and then examines each pixel whether it is part of noise or part of signal. Final decision about each pixel is taken according to the majority verdicts about the pixel. Values of the corrupted pixels are found by fitting a general paraboloid only through the uncorrupted pixels in each window and taking the mean of all suggested values. The method can be applied to a vast area of real world applications like digital photography, medical imaging, satellite imagery correction, computer vision and so on. Experiment reveals that this method has success rate of more than 92% in detection and elimination of impulse noise.

Keywords: Edge Pixel, Impulsive noise, Least Squares Regression, Noise Density.

I. INTRODUCTION

Noise Reduction in digital images is an interesting topic for researchers. As some sort of error is always associated with signal handling devices, restoring the real signal from noise becomes a challenging task.

Due to several reasons e.g. Inappropriate Acquisition, Analog to digital conversion, Quantization, exposure setting, Camera ISO speed, scanning problem, dust in lens, film grain etc. random noise can be initiated in digital images leading image quality degradation [1],[3]. The process of minimizing the noise and thus presenting a lucid image is a necessary objective of myriad research fields related to Remote image acquisition, Satellite Imagery, Digital Photography, Image Processing and Understanding, Medical Imaging, Computer Vision [1].

Mathematically, the goal is to produce the best possible estimate $\hat{a}[m,n]$ of the original image $a[m,n]$ from a recorded image $c[m,n]$. The measure of success is usually an error measure between the original $a[m,n]$ and the estimate $\hat{a}[m,n]$ as no mathematical error function is known that corresponds to human perceptual assessment of error [1].

Depending on several parameters, image noise can be classified in two groups - dependent (Gaussian) or independent (random). We address here one kind of random noise, also known as salt and peeper noise [7], which appears on the image as additive random impulsive dots or small regions.

In natural images, to distinguish the distorting pixel from the real one is an ill posed problem since it is not always known or well defined which measures actually answers whether a pixel is corrupted or not [5].

Most of the solutions on this problem work under certain assumptions, conditions and using some priori knowledge about the noise involving phenomenon. [2]

Our basic assumptions here are:

1. Noise is distributed throughout the whole image. There is a pre-assumed noise density, which tells about how dense the corrupted pixels are.
2. Noisy pixels vary more than a threshold value with most of the surrounding pixels.

In this paper we propose a method to determine the impulsive noise using window based local statistics and regression analysis. To decide whether a pixel is corrupted or not, the method uses up to s^2 judgments for a single pixel. Each judgment is done by fitting a general plane on a square window of dimension $s \times s$, which outputs its verdicts about each of the $s \times s$ pixels inside. Moreover, it is difficult to discriminate between and edge and an impulse. Strength of our method is, chance to detect an edge pixel as impulsive noise is very low, because final decision about a pixel is made by combining the output of up to s^2 judgments. To get the estimated value for the corrupted pixel we have used a paraboloid approximation per window taking only good pixels in consideration, because this partly supports the gradual rise and fall nature, which is perfectly suited for natural images having continuous tone.

II. BACKGROUND

If several images of same object differing only in noise realization are available then temporal averaging reduces the noise [1]. When temporal averaging is not possible, methods developed from linear filtering or smoothing is sometimes used.

Early techniques involved general statistical ideas or frequency domain concept [1],[2].

Lee [3] describes methods for contrast enhancement and white noise filtering of a digital image based on local statistics.

Frequency based noise reduction is more or less established on the fact that, a small window in a continuous tone image contains less high frequency components. So, controlling high frequency components results in reduction of noise.

But these blind methods don't take the position of the error into account. So directly applying these methods often leads to undesired decrease in image sharpness. Applying a low or mid pass filter often makes the image blurred at edges and causes loss of information. Young *et al.* [1] compares several of these techniques.

A classical solution in this regard is Wiener filter [1] which works with apriori noise and power spectra. But, as the noise and power spectra are unknown in most of the cases, several extensions or alternatives exist considering fully or partly frequency based statistics and wavelet decomposed methods [6]. Research works also included Fractal based methods or combining Fractal, wavelet and frequency based methods [5].

Modern techniques on this topic use algebraic manipulations. Introductory concepts of image restoration based on algebraic methods, Zero order approximation, Bilinear Interpolation for Gray Level Thresholding, constrained and unconstrained Least squares are discussed in [2].

In most of the cases the measurement of error is done by Least Squares method. Though Rudin *et al* [8] emphasizes on using L_2 norm, gain in performance is not that high compared to the extra overhead needed. To avoid massive amount of calculations we use L_1 norm for the Least squares criteria in the paper.

Moreover, Eng and Ma [4] present a scheme for impulse noise detection and denoising. Their system decides the corrupted pixels first by applying median filter of certain window size determined by the noise density. By applying fuzzy set rules, the system discriminates between edge pixel and non-isolated impulse noise, and then denoises the image using median filter or FWM filtering.

However, these methods suffer from a common problem of extensive computation or iterative denoising, which is not feasible many cases, especially when handheld devices are involved and real time processing is necessary. Hence, existing methods are not easily employable in small handheld devices like mobile or handicam, with low power operation facility.

III. DESCRIPTION

Our proposed method works basically in two stages.

Stage 1 : Detect the pixels which are corrupted.

Stage 2 : Keep the uncorrupted pixels intact. Estimate values for the corrupted pixels from its neighboring good pixels.

Throughout the discussion, let W be an image of dimension $m \times n$ corrupted by impulsive noise of known average strength and density. Let the value of a pixel at (u, v) position of the image = $W(u,v)$

A. Noise Detection

For each position *(x,y)*, $1 \le x \le m-s+1$ $1 \le y \le n-s+1$ in the image, our method processes a square window of dimension $s \times s$ stretching from (x,y) to $(x+s-1, y+s-1)$, where s is provided by user. s mainly depends on noise density.

Table I Suggestion for Window Dimension s

Noise Density	Dimension
$0 < \rho \le 0.3$	4×4
$0.3 < \rho \le 0.6$	6×6
$0.6 < \rho \le 0.8$	8×8

A.1 Plane Approximation for windows

If (x,y) is the starting point of a window $W_{x,y}$ and (i,j) $0 \le i, j \le s-1$ is the offset of a pixel, then $(i+x, j+y)$ is the global position of that pixel.

For each window, the method first fits a plane with the pixel-values in the window by Multiple Linear Regression.

General Equation of a plane inside the window starting at *(x,y)* is: $Z_{x,y}(i,j) = c_1 + c_2 i + c_3 j$

For a Least squares fit, the linear system formed is $A \times C_{x,y} = B_{x,y}$

$$\text{where, } A = \sum_{i=0}^{s-1}\sum_{j=0}^{s-1}\begin{bmatrix} 1 & i & j \\ i & i^2 & ij \\ j & ij & j^2 \end{bmatrix}$$

$$B_{x,y} = \sum_{i=0}^{s-1}\sum_{j=0}^{s-1}\begin{bmatrix} W(i+x, j+y) \\ i.W(i+x, j+y) \\ j.W(i+x, j+y) \end{bmatrix}$$

and $C_{x,y} = \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix}^T$

Matrix A depends only on window size. For s = 4, calculated value of A is :

$$A = \begin{bmatrix} 16 & 24 & 24 \\ 24 & 56 & 36 \\ 24 & 36 & 56 \end{bmatrix}$$

A.2 Local Classification

From the plane determined for the window starting at *(x,y),* tentative value of $Z_{x,y}(i,j)$ $0 \le i, j \le s-1$ is calculated. Plane Pixel Deviation for each offset *(i,j)*

$$PPD_{x,y}(i,j) = \left| Z_{x,y}(i,j) - W(i+x, j+y) \right|$$

To find the most of the corrupted pixels our method depends on two external input parameters.

Deviation Parameter δ = Maximum Plane Pixel deviation that let the window render it uncorrupted.

Density Parameter ρ = Average density of corrupted pixels. It is assumed that corrupted pixels are distributed everywhere in the image $\rho \in [0,1]$. Experiment shows, taking a slightly larger value for Density parameter, than the assumed Noise Density yields better result.

If $PPD_{x,y}(i,j) > \delta$ current window reports the pixel at offset (i,j) as corrupted. If total number of reported corrupted pixels in a window exceeds $s^2\rho$, judgment of that window is not accepted.

For example, if s=4, suppose a window:

$$W_{u,v} = \begin{bmatrix} 40 & 52 & 55 & 58 \\ 60 & 62 & 90 & 60 \\ 5 & 70 & 60 & 58 \\ 55 & 61 & 64 & 25 \end{bmatrix}$$

$C_{u,v} = [52.14 \quad \text{-}1.98 \quad 3.68]$ calculated from $W_{u,v}$.

$$Z_{u,v} = \begin{bmatrix} 52.14 & 55.81 & 59.49 & 63.16 \\ 50.16 & 53.84 & 57.51 & 61.19 \\ 48.19 & 51.86 & 55.54 & 59.21 \\ 46.21 & 49.89 & 53.56 & 57.24 \end{bmatrix}$$

$$PPD_{u,v} = \begin{bmatrix} 12.14 & 3.81 & 4.49 & 5.16 \\ 9.84 & 8.16 & 32.49 & 1.19 \\ 43.19 & 18.14 & 4.46 & 1.21 \\ 8.79 & 11.11 & 10.44 & 32.24 \end{bmatrix}$$

Taking δ =25, pixel at offset (1,2), (2,0) and (3,3) are reported corrupt by the current window as PPD there exceed 25.

A.3 Global Classification by Majority Vote

Final decision about a pixel is taken according to majority vote i.e. verdict of most of the windows about it.

After processing the windows is complete, for each pixel *(u,v)* we have,
J(u,v) = number of accepted judgments for pixel at (u,v)
R(u,v)= number of verdicts reporting pixel at (u,v) uncorrupted $\alpha(u,v) = \dfrac{R(u,v)}{J(u,v)}$, when $J(u,v) \neq 0$

Verdict of a window is not accepted when noise density reported there exceeds the assumed density parameter ρ. This may happen in the region where the image has high contrast grainy parts or near some sharp edge of the object as there is an abrupt change in pixel value. So the plane approximation may misclassify some edge or object boundary pixels and noise presented there, especially in case of high noise.
To discriminate between edge and noise we introduce:
Classifier Parameter, $\Omega \in (0,1)$

Pixel at (u,v) is assumed belonging to Edge part or boundary region of some object in the image if $J(u,v) < \Omega s^2$, i.e. in case of the pixels near edges, a certain fraction of the windows yield unreliable answer. Value of Ω depends on sharpness of the image.
$\Omega = 0.4 \sim 0.5$ for natural images.
Threshold Ratio, φ = Minimum $\alpha(u,v)$ needed for a pixel to be declared uncorrupted globally. $\varphi \in (0,1)$
We define two Threshold ratios,
φ_e : used for edge pixel or grainy part
φ_n : used for flat or non grainy regions
If s = 4 value of $\varphi_e = 0.6 \sim 0.75$ and $\varphi_n = 0.75 \sim 0.9$.

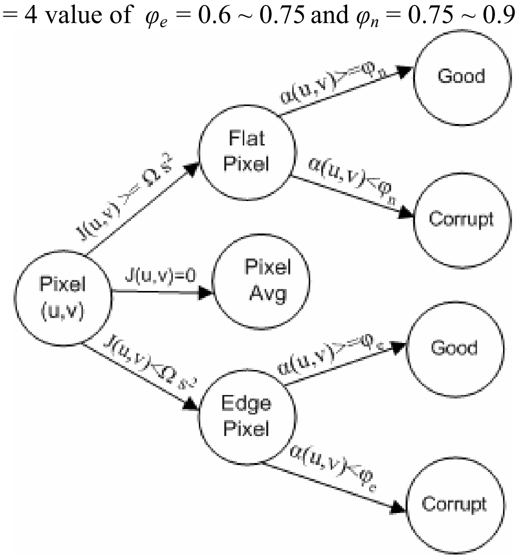

Fig. 1 Decision Tree to decide about a pixel

When $J(u,v) = 0$ the pixel is neither corrupted nor good. But in the next phase, during filtering, it is applied an average of the neighboring values. Fig 1 summarizes the whole process of classification.

B. Noise Filtering

The values of the corrupted pixels are estimated from the values of nearby uncorrupted pixels. As we are dealing with impulsive noise, which change the pixel value too much, the value of a noisy pixel is not taken into account. Let, $\hat{W}(u,v)$ be the estimated value for corrupted pixel *(u,v)*.

B.1 Paraboloid Approximation

For each window of dimension $s \times s$ at *(x,y)* the method fits a general paraboloid $K_{x,y}$ through the uncorrupted pixels inside that window.

$$K_{x,y}(i,j) = c_1 + c_2 i + c_3 j + c_4 i^2 + c_5 j^2 + c_6 ij$$

If $C_{x,y} = [c_1 \quad c_2 \quad c_3 \quad c_4 \quad c_5 \quad c_6]^T$,
$A \times C_{x,y} = B_{x,y}$ is solved for *C*.

$$B_{x,y} = \sum_{i=0}^{s-1}\sum_{j=0}^{s-1}\left(\begin{bmatrix} w(i+x,j+y) \\ i.w(i+x,j+y) \\ j.w(i+x,j+y) \\ i^2.w(i+x,j+y) \\ j^2.w(i+x,j+y) \\ i.j.w(i+x,j+y) \end{bmatrix}, \ w(i+x,j+y) \text{ is uncorrupted}\right)$$

$$A = \sum_{i=0}^{s-1}\sum_{j=0}^{s-1}\left(\begin{bmatrix} 1 & i & j & i^2 & j^2 & ij \\ i & i^2 & ij & i^3 & ij^2 & i^2 j \\ j & ij & j^2 & i^2 j & j^3 & ij^2 \\ i^2 & i^3 & i^2 j & i^4 & i^2 j^2 & i^3 j \\ j^2 & ij^2 & j^3 & i^2 j^2 & j^4 & ij^3 \\ ij & i^2 j & ij^2 & i^3 j & ij^3 & i^2 j^2 \end{bmatrix}, w(i+x,j+y) \text{ is uncorrupted}\right)$$

Due to absence of information in corrupted pixels A may be singular. In that case a paraboloid approximation is not possible and a Plane approximation $K_{x,y}(i,j) = c_1 + c_2 i + c_3 j$ is taken.

B.2 Local Suggestion

From the regression coefficients, the window at *(x,y)* suggests for each corrupted pixel $\hat{w}(i+x, j+y) = K_{x,y}(i,j)$

B.3 Global Estimation

For each corrupted pixel *(u,v)* we have,
S(u,v) = sum of suggested values for *(u,v)*
T(u,v) = number of suggestions for *(u,v)*

Final Estimation, $\hat{W}(u,v) = \dfrac{S(u,v)}{T(u,v)}$, when $T(u,v) \neq 0$

If $T(u,v) = 0$ or $J(u,v) = 0$, value of that pixel is determined by averaging the neighboring pixels, after final estimation of other corrupted pixel is complete.

IV. SIMULATION

Performance of the method can be calculated only by simulation when the error is known. Simulation is done with standard test image 'lenna' artificially corrupted by Noise Strength approximately 50. When noise density is low or medium result seems better for window dimension 4×4, and discrimination between edge pixels and noisy pixels is also better.

A. Noise Detection

Effect of several parameters on performance of noise detection was examined.
We define Performance of the Noise Detection part by two functions:

Classification Efficiency,
$$= \frac{\text{Number of Pixels classified correctly}}{\text{Total number of Pixels in the image}} \times 100\%$$
Error Detection Efficiency,
$$= \frac{\text{Number of corrupted pixels correctly detected}}{\text{Total number of corrupted pixels}} \times 100\%$$

A.1 Effect of Deviation Parameter

The effect of deviation parameter for noise density 26% is shown in Fig 2 and 3.
$\varphi_e = 0.7$ and $\varphi_n = 0.85$, $\rho = 0.34$, $\Omega = 0.5$, $s = 4$
Highest performance is found at δ=30, Detection Efficiency about 87% and Classification Efficiency about 91%.

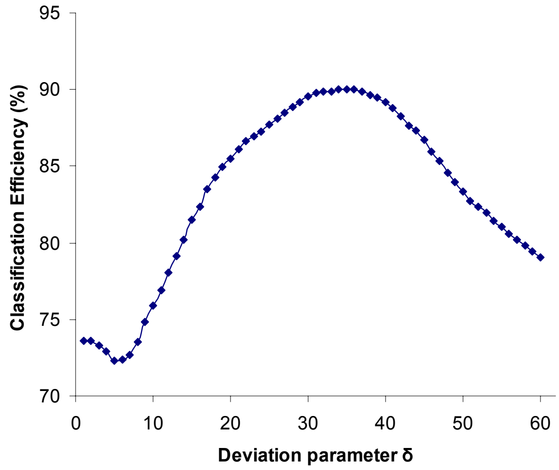

Fig. 2 Effect of Parameter δ in Classification Efficiency

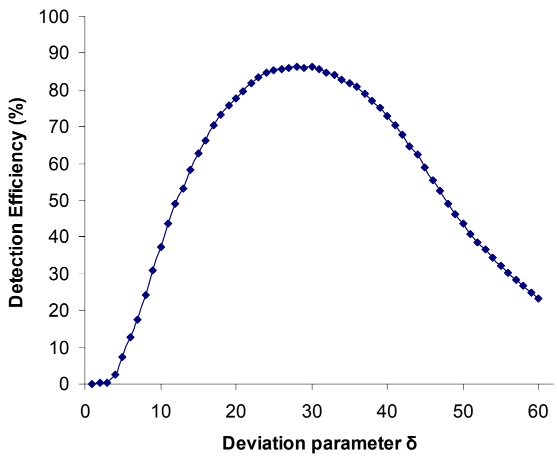

Fig. 3 Effect of δ in Detection Efficiency

A.2 Effect of Density Parameter for fixed Noise Density

For noise density 30% optimal value of ρ is 0.4 as depicted in Fig. 4.

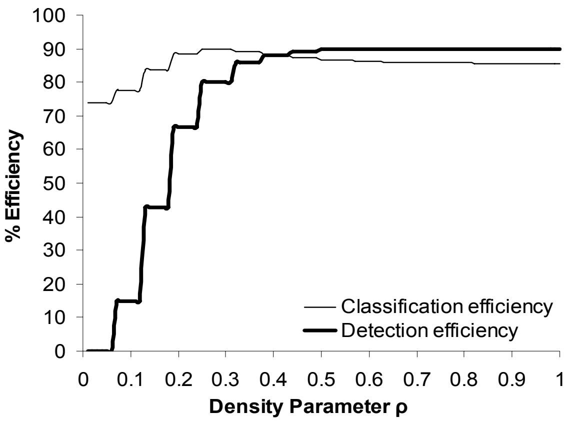

Fig. 4 Effect of ρ in Efficiency for Noise Density 30%

A.3 Effect of Threshold Ratio

With decreasing Threshold Ratio, Error Detection Efficiency decreases and Classification Efficiency increases (Fig. 5) Reason is, as we try to include all the noise, some of the uncorrupted pixels are also treated as noise which decreases Classification Efficiency, and vice versa. So there is a tradeoff between these two.

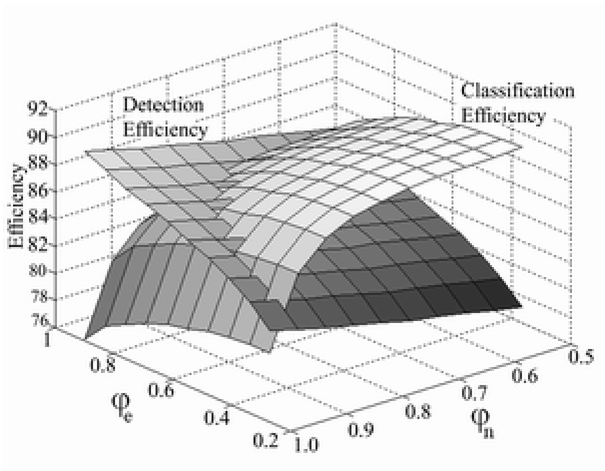

Fig. 5 Effect of φ in Efficiency

For ρ = 0.4, Ω = 0.5, s=4, Noise Density = 30%, optimal value of φ_e = 0.7 and φ_n = 0.85.

A.4 Performance at various Noise Density

Noise Detection was performed for various levels of noise in the image shown in Fig. 6 and Fig. 7.
Parameters: δ=30, φ_e = 0.7 and φ_n = 0.85, Ω = 0.5
Classification Efficiency is 98% at noise level 6%. It decreases as noise increases. At noise level 55% it becomes 77%.
Noise Detection is most efficient (88%) at noise level 10% to 40%. But the efficiency doesn't fall that much as noise in increased. At noise level 55% Detection efficiency is 83%.

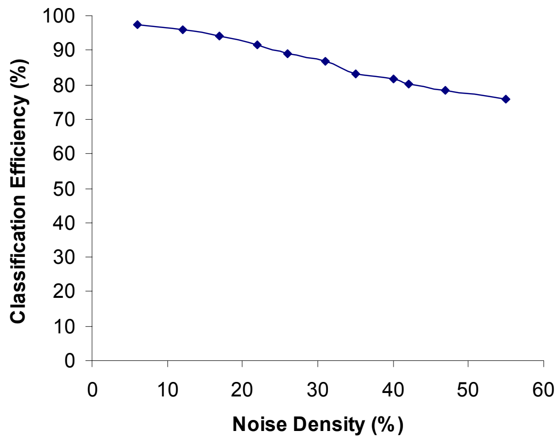

Fig. 6 Effect of Noise Density in Classification Efficiency

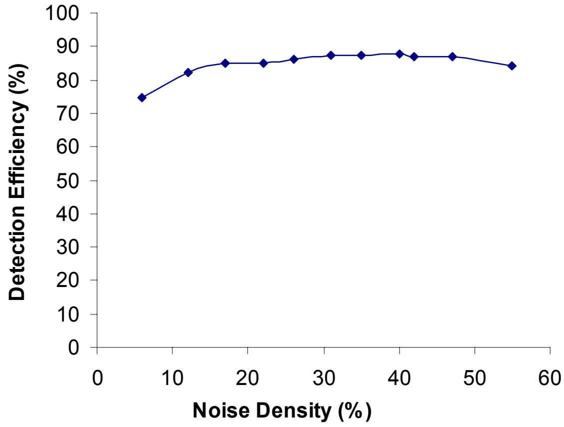

Fig. 7 Effect of Noise Density in Detection Efficiency

B. Noise Filtering

Noise Filtering i.e. estimation of the values of corrupted pixels is best when noise density is low to medium. Here we measure filtering performance by Peak Signal to Noise Ratio (PSNR) shown in Fig. 8. Output for randomly generated 6%, 12% and 30% noisy image Fig. 9(a), Fig. 10(a), Fig. 11(a) are shown in Fig. 9(b), Fig. 10(b), Fig. 11(b) respectively. For Noise Density >40%, the output suffers from information loss, as a great amount of information is to be estimated.

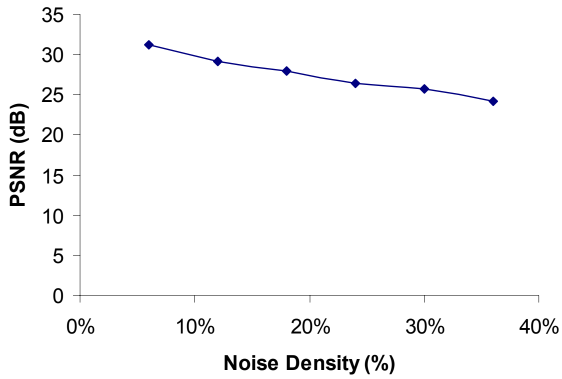

Fig. 8 Peak Signal to Noise Ratio vs. Noise Density

Fig. 9(a) 6% corrupted image

Fig. 9(b) Output at 6% noise

Fig. 10(a) 12% corrupted image

Fig 10(b) Output at 12% noise

Fig.11(a) 30% corrupted image

Fig.11(b) Output at 30% noise

V. COMPLEXITY ANALYSIS

Number of windows $= (m-s+1)(n-s+1)$

Cost per window for Local classification: O(s^2)

Time for Global Error Classification : O(mn)

Filtering : O(ρs^2) per window

Final Estimation : O(mn)

Total Cost :

$$O((m-s+1)(n-s+1)s^2 + mn + \rho s^2(m-s+1)(n-s+1)+mn)$$
$$= O(mns^2(1+\rho))$$

VI. OUTCOME OF THE RESEARCH

The method performs best in noise density < 40%, which is a good limit for natural images.

Main successes of the method are:

1. It doesn't use blind mean or median filtering here, so the output doesn't suffer from unwanted loss in sharpness.
2. No complicated mathematical operations or transformation are needed and also specialized data structure is not necessary.
3. Implementation logic is easy and economical with resources.

Hence, our method is eligible for integration in digital camera and other handheld or embedded devices.

Shortcomings of the method are:

- Noise detection is done in single pass, and filtering is also done in another single pass. Multilevel detection and filtering would improve performance.
- For Regression, L_1 norm is used. Less calculation is needed, but it results in less accuracy.
- Present method only concentrates in algebraic methods. Considering frequency information and wavelet based statistics along with, would yield better result in noise detection and removal.

VII. CONCLUSION

We described an easy to implement method to detect impulsive noise successfully from natural images and eliminate them. The method is simple yet powerful and can be extended to other types of noise also. As the detection system is based on multiple judgment and verdicts of the windows employed, and the restoration system is based on multiple suggestions about the tentative value of a pixel, it is context aware and less prone to misclassification and loss of sharpness.

Combining the present method with frequency based methods may give better estimation of the actual value of the corrupted pixel. The majority vote detection system employed here can be equipped with Artificial Intelligence. We target to stack the scheme on a neural network based architecture, involving "experienced" and "learned" agents for further highly sophisticated and careful reduction of all type of noise to minimum.

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